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Computation of RATAN-600 characteristics in operation as a "radio–Schmidt telescope"

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Abstract. Results of calculation of characteristics of the radio telescope RATAN-600 operated as a "radio–Schmidt telescope" are presented. The flat reflector and the circular mirror are computed as a double-mirror aplanatic system with a planoid mirror which makes "Schmidt" correction of the wave-front of the wave incident at an arbitrary angle with respect to the horizon. Formulae are derived and a package of programmes is created for calculation of configurations of the mirrors of the antenna system and optimization of its basic parameters. Phase distribution of the field over the aperture and the beam pattern (BP) of the telescope for different angles of formation of the antenna and different elevations of a source are computed. A possibility is shown of long-time (up to 1 hour) tracking of sources at waves longer than 4 cm with an aperture of 150 m.

Key words: telescopes: radio characteristics

Long tracking of space sources at the radio telescope RATAN-600 is currently one of the central problems. Its solution will make it possible to improve the sensitivity of the telescope and widen the scope of astronomical tasks performed with RATAN-600. The mode of long tracking is especially vital to spectral investigations, to study of fast variable space objects such as pulsars, to investigation of dynamics of solar processes and radio scintillations. The use of the flat periscopic reflector located in the southern part of the radio telescope is most efficient for long tracking of sources (Khaikin et al., 1972; Shivris, 1980). The flat mirror in combination with the south sector of the main mirror and the secondary mirror made in a form of a parabolic cylinder with a horizontal generant represents a system similar to that of Kraus in Nancay (Fig. 1).

There is an arch rail track between the flat mirror and south sector of the main mirror to enable movement of the secondary mirror. In the traditional approach long tracking of a cosmic source requires an appropriate tilt of the flat mirror panels, movement of the feed on the arch rails and continuous setting of main mirror panels in azimuth and radius so that the mirror would take shape of a parabolic cylinder with a vertical generant, which is placed in accordance with the azimuth of the wave front reflected from the periscope. This most powerful mode ("running parabola"), when implemented, will make possible tracking of sources for 3–4 hours with a full antenna aperture. However, this will demand essential upgrading of the automatic control system of RATAN-600.

Long tracking of a cosmic source with no or minor reduction of the aperture is also possible in the modes proposed by N. L. Kaidanovsky and other authors. The mode with the main mirror shaped as a circular cylinder and a special double-curvature feed, the "circular telescope" etc. are among them. These modes, however, have not yet been implemented because of design difficulties, since they require a special correcting mirror of intricate shape (Vavilova, 1969), use of a phased antenna array in the focal line of the secondary mirror (Kaidanovsky, 1980), additional rail tracks (Kaidanovsky, 1975).

Tracking of sources with RATAN-600 is presently done with the aid of the "sliding" (Mingaliev et al., 1985) and "relay" (Golubchina, 1986) methods. A mode of "fast setting" of the main mirror has been developed and tested (Khaikin et al., 1997). The "relay" condition enables tracking of a cosmic source for 6–10 hours, but it narrows the antenna aperture by a factor of 5–6 or the frequency band when "zoning" the main mirror. The other two techniques can be operated with a full antenna aperture, but the exposure time is short (1–10 min).

The technique which is suggested to be discussed in the present paper and called "radio–Schmidt telescope" employs the antenna system "South+Flat". To follow a source, the secondary mirror moving on the arch rails is used and a change of the tilt angle of the flat mirror within small limits is made, the

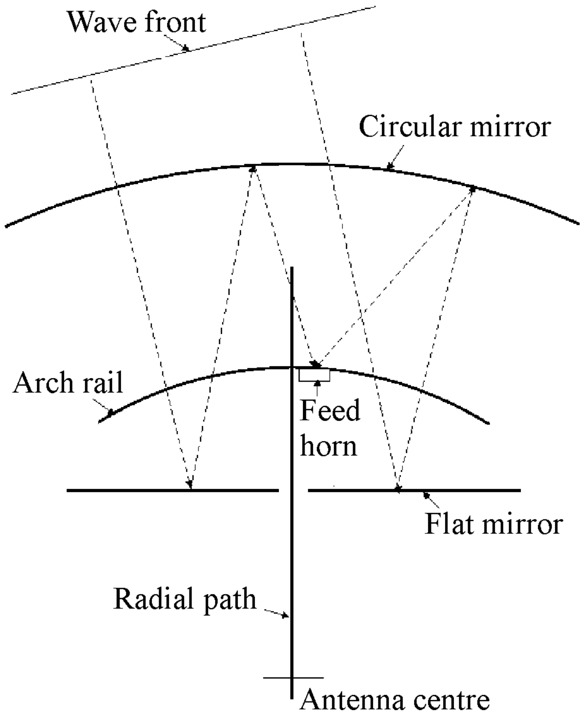

Figure 1: *A scheme of the RATAN-600 radiotelescope ("South sector + Flat reflector").*

same as under the condition of "running parabola". A distinguishing feature of this procedure is that the main mirror shape close to a circular cylinder does not change in the course of source tracking.

The method is evolved from the idea of creation of a prefocal aplanatic double-mirror system with a wide field of view, free from spherical aberration and coma. The flat and circular mirrors form the basis of the system. It will be recalled that a prefocal system in optics is one in which a beam of light striking the first mirror is reflected from it and sent to the second mirror without crossing the optical axis of the system.

In optics, to such systems belong the system of Schwarzschild, Ritchey–Chretien, "mirror Schmidt" and "mirror Wright". The latter two are a particular case of double-mirror aplanatic systems — systems with a planoid mirror. The surface curvature in planoids at the vertex equals zero, so the focal length of the system is approximately equal to that of the second mirror. In the case of the antenna system "South+Flat" the planoid is the flat mirror, the circular antenna of RATAN-600 is the second mirror. Note that the idea of "Schmidt" correction of the wave front for a radio telescope with an immovable circular main mirror was first advanced by Arsac and realized in the system of Kraus in Nancay (Arsak, 1960, Biraud, 1969).

1. The design of the antenna system at $H_a = 0°$

The theory of prefocal aplanatic systems of two mirrors in optics was developed by K. Schwarzschild, Chretien, D. D. Maksutov. In the 1930s, Wright proposed and computed a design of a shortened Schmidt telescope. One of the disadvantages of the Schmidt system — the necessity of placing the correcting mirror at a double distance from the focus of the spherical mirror at the centre of its curvature — was eliminated through the replacement of the spherical mirror by an oblate spheroid and placing the correcting mirror at a distance close to focal. Wright's system is an aplanatic system in which, apart from spherical aberration, coma is also removed. This system, however, has a smaller (1.5–2 times) field of view as compared with Schmidt's.

Since the formulae derived by Wright (1946) are in no way restricted by the dimension of a telescope and can be applied to calculation of systems of any size, we used them for preliminary modeling of the shape of the circular and flat mirrors of the system ("South+Flat") under the condition "radio–Schmidt telescope". It is shown in Fig. 2 how the shape of the flat and circular mirrors changes with variation of parameter $(F' - M)$, where F' is the paraxial focal distance of the entire system, M is the distance of the circular antenna vertex from the focus where the feed is placed. $\Delta x = x(y) - x(0)$ is the variation of the flat mirror longitudinal coordinate, r is the radial coordinate of the circular reflector (Fig. 3).

Preliminary calculations show, firstly, that it is possible to construct an aplanatic system based on the combination "South+Flat", which is similar to those of Schmidt and Wright, secondly, they demonstrated the necessity of optimization of its parameters to attain the utmost efficiency of the telescope. In further calculations of the double-mirror aplanatic system, we used the method proposed by Popov (1974), which is based on examination of differential equations whose solution will make it possible to find accurate expressions for mirror surfaces being surfaces of rotation and having a common axis of symmetry. The development of the formulae obtained by Popov (1974), Popov et al. (1976) and presented below was based on the condition of producing an image free from spherical aberrations on the axis (principle of Fermat) and on the condition of aplanatism (Abbe sine condition).

Although the flat and circular reflectors of the radio telescope RATAN-600 are not surfaces of rotation, the expressions derived in the above papers can

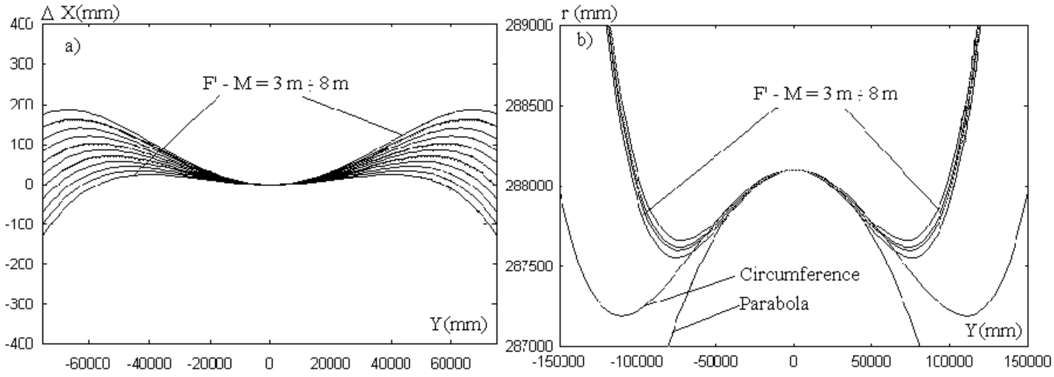

Figure 2: *Shape variation of the flat (a) and circular (b) reflectors with different values of parameter $F' - M$.*

be used to calculate the curves needed to set the mirror panels, when the position angle of the front of the incident wave H_a is equal to zero. This is associated with the characteristic properties of focusing of the rays in the system "South+Flat". At elevations equal to zero, in approximation to geometric optics, the ray path in the horizontal sections of such a system coincides completely with that of the meridional section of the mirrors having the axis of rotation, which is explained by propagation of the cylindrical wave and focusing of the rays only in the horizontal plane. The geometrical dimensions of the antenna system "South+Flat" are shown in Fig. 3a. D, the distance between the vertices of the mirrors, is 184 m, Q, the distance between the feed on the arch rails and the flat mirror vertex, is 52 m, M, the distance between imaginary secondary mirror and the vertex of the circular antenna or the focal distance of the circular antenna, is 132 m.

Now present the expressions derived in (Popov and Popova, 1976). The paraxial distance f' of the system was taken equal to 1.

The coordinates (x, y) of the flat mirror and the radius-vector (ρ) of the circular reflector can be found by the formulae:

$$x = \frac{\xi(1-\xi)}{d} - d + \frac{d+q}{d^{(2d-1)/(d-1)}} \cdot \cdot (1-\xi)^{1/(1-d)} (d-\xi)^{(2d-1)/(d-1)}, \quad (1)$$

$$y = 2\sqrt{\xi(1-\xi)}, \quad (2)$$

$$\frac{1}{\rho} = \frac{\xi}{d} + \frac{d^{1/(d-1)}}{d+q} \frac{(1-\xi)^{d/(d-1)}}{(d-\xi)^{1/(d-1)}}, \quad (3)$$

where $\xi = \sin^2(\psi/2)$.

The origin of Cartesian coordinates (x, y) and the pole of the polar system (ρ, ψ) are at the focus F (Fig. 3b), the distance q of the focus from the vertex of the mirror, which is the first in the ray path, is positive if the focus of the system is on the right relative to the mirror (flat in our case) and negative if the focus is on the left. Quantity d is always positive. $d = D/F', q = Q/F', m = M/F'$. With $F' = M$, $d = 1.34$, $q = -0.34$, $m = f' = 1$.

The circular reflector of the radio telescope RATAN-600 consists of individual panels 11 m×2 m in size which can be moved along the radius within ±0.5 m from the main circle of radius $R_a = 288$ m; the range of displacements in azimuth is ±6°, and 0° ÷ 50° in position angle. In the standard mode the flat mirror panels (8 m×3 m) can be moved only in position angle (0° ÷ 50°). They can change their position in longitudinal (meridional) direction only with the aid of special adjustment devices by no more than 200 mm along the longitudinal coordinate and by 2° in azimuth.

The basic criterion in the computation was the creation of such configurations of the mirrors that could enable a maximum number of panels of the flat and circular reflectors to be set up, i.e. attaining a maximum effective surface and the longest time of following a cosmic source. The effective surface is first of all determined by the number of the flat mirror elements set up, while the time of tracking by the number of the set-up circular reflector panels. With the values of D, Q and M of the system "South+Flat" fixed, this can be done by varying the value of the paraxial focus F' of the whole system. From the condition that the system is planoid, it follows that the values of F' must not be much different from the focal distance M of the circular reflector.

We computed families of curves $x = f(y)$ and $\rho = f(\psi)$ with different sizes of the flat mirror $L = 2R_a \mathrm{tg}(\varphi_0)$, which illustrate the way the shape of the flat and circular reflectors changes in the horizontal plane with varying paraxial focus F' values, ($2\varphi_0$ is the angle at which the aperture of the flat mirror is visible from the centre of the antenna O). By the method of sequential approximations for different values of φ_0 the optimum paraxial focus F'_{opt} would be found, at which the variations of the coor-

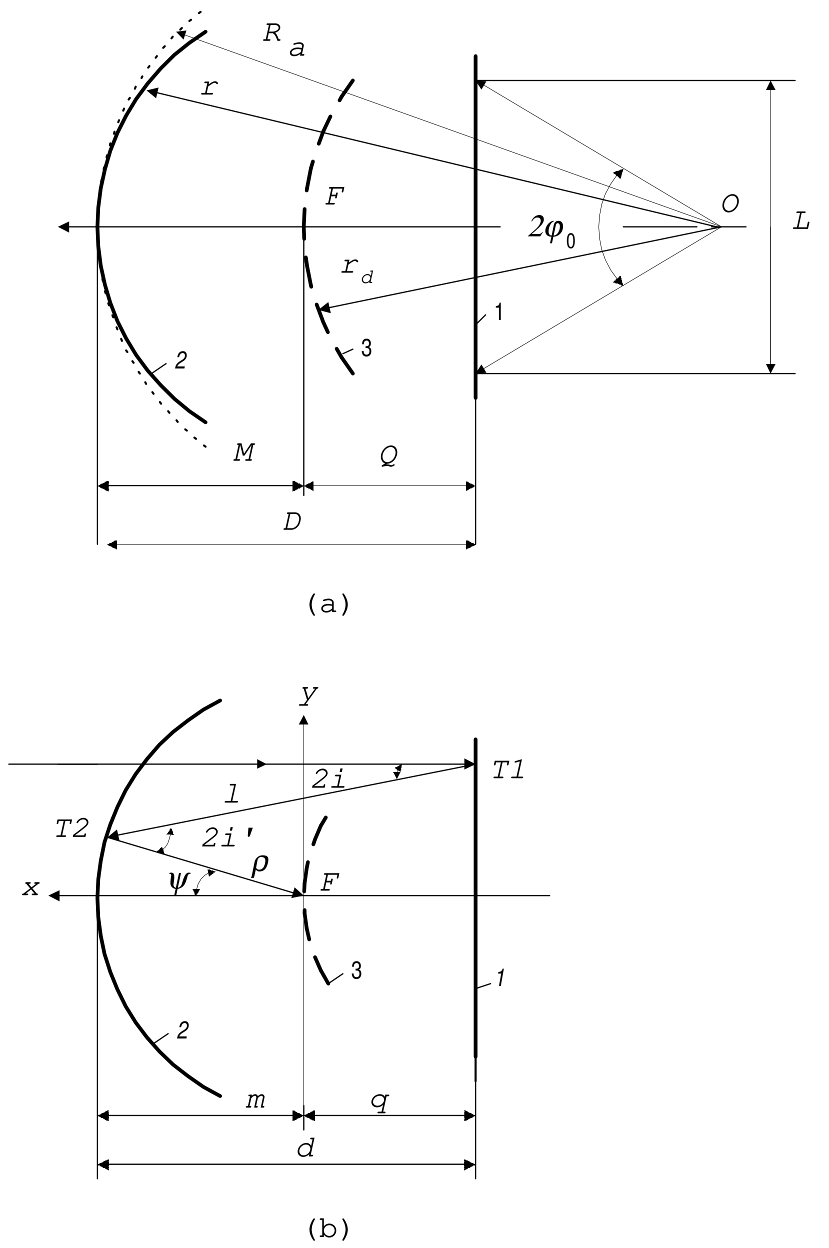

Figure 3: *a): geometrical parameters of the radiotelescope periscopic system "South + Flat", b): ray path in the "radio-Schmidt telescope" mode at $H_a = 0°$ and $\alpha = 0°$.*

dinate x were minimum, i.e. the minimal longitudinal displacements of the flat mirror panels.

Fig. 4 presents the relationships $\Delta x_{max} = f(\varphi_0)$ and $\Delta r_{max} = f(\varphi_0)$ derived at optimal values of the paraxial focus. Δx_{max} is the maximum longitudinal displacement of the flat mirror element, $\Delta r_{max} = (R_a - r)$ is the maximum radial shift of the circular reflector elements at a given value of angle φ_0.

Using the relations displayed in Fig. 4, one can estimate the maximum horizontal size of the flat mirror, at which the longitudinal displacements of its elements do not fall outside permissible limits of their movement. This size is 150 m – 170 m ($\varphi_0 = 15° \div 17°$). The maximum circular reflector aperture with which the radial displacements of its panels are held within allowable limits is $\varphi_0 \cong 20°$, which corre-

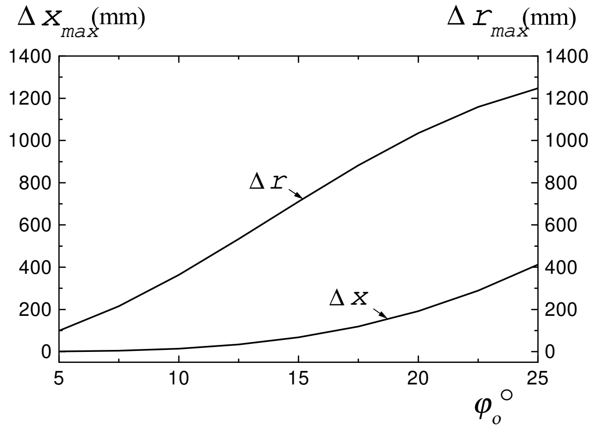

Figure 4: *Maximum longitudinal displacement of the flat mirror elements, Δx_{max}, and maximum radial shift of the circular reflector elements, Δr_{max}, versus angle φ_0.*

sponds to ≈ 200 m of the horizontal aperture.

The computations discussed refer to the case where the feed is located between the flat and circular mirrors at a distance $M = 132$ m from the vertex of the latter and can be moved on the arch rails whose radial distance from the antenna centre is $r_d = 156$ m.

Applying formulae (1–3), the configurations of the mirrors with varying M, i.e. with changing the location of the feed between the vertices of the flat and circular reflectors, were also computed. The distance between the mirrors did not change. The relations $\Delta x_{max} = f(M)$, $\Delta r_{max} = f(M)$ for $\varphi_0 = 10^\circ$ and 15° are exhibited in Fig. 5.

The calculations have shown that the maximum longitudinal shift of the flat mirror panels decreases as the feed approaches the flat mirror ($M = 184$ m), while the maximum radial displacement of the circular reflector elements is a minimum at $M = 144$ m. Thus, the current radius of the arch rails is not optimal. From the point of view of the longest tracking of the target, an arc of a circle with a radius of 144 m seems to be optimal.

2. Computation of the field phase distribution in the aperture and in the BP of the antenna system "South+Flat" when observing in azimuths at $H_a = 0^\circ$

We will now imagine that the surfaces of the mirrors are composed of an infinite number of elements whose horizontal sizes tend to zero. The principal parameters of the system are as follows: $D = 184$ m, $M = 132$ m, $r_d = 156$ m. The surface shape of the mirrors is described by expressions (1–3). These were derived for the case where the flat front of the wave incident on the first mirror makes an angle $H_a = 0^\circ$ with the horizon and an azimuthal angle $\alpha = 0^\circ$ with the antenna system axis (passage of a target across the meridian). In this case, the sum of optical paths from the focus to any plane normal to the optical axis is constant, and the field is distributed in the aperture in phase, provided that the elements are set in rigorous conformity with the computed curves.

Calculate what the phase field distribution over the aperture will be like when the wave front is incident at α different from zero, and the feed is displaced on the arch rails away from the system axis. In this case the position of a new focus, F_d, in the polar coordinate system, the origin of which, O′, is located on the system axis at a distance p from the centre of the circle O, is defined by coordinates (r_d, α) (Fig. 6). The sum of optical paths, S, from the new focus F_d to the plane A, which is perpendicular to the propagating wave front and goes through the flat mirror center, is:

$$S = t + l + \rho' , \tag{4}$$

where t is the distance of the plane A from the point T_1 on the flat mirror, l is the separation of points T_1 and T_2 on the surface of the flat and circular reflectors, ρ' is the distance of point T_2 on the circular reflector surface from the focus F_d or the radius-vector of point T_2 in the polar coordinate system (ρ', ψ').

Calculate the phase field distribution over the aperture expressed in a linear measure $\Delta S(\psi') = S(\psi') - S(0)$.

$$t = y_1 \sin\alpha - x_1 \cos\alpha \, . \tag{5}$$

The coordinates x_1, y_1 of point T_1 were computed by formulae (1, 2), l is described by the equation of a straight line going through point T_1 with coordinates (x_1, y_1) and having the tilt angle $(\alpha - 2i)$ with respect to the y axis:

$$y_1 - y = (x_1 - x)\,\mathrm{tg}(\alpha - 2i) , \tag{6}$$

where i is the angle, which a line tangent to the curve, the flat mirror elements are aligned on, makes with y axis

$$i = i' - \psi/2 , \quad \mathrm{tg}\, i' = \frac{1}{\rho}\frac{d\rho}{d\psi} .$$

The ray reflected from the flat mirror intersects the circular mirror at point T_2 with coordinates ($x_2 = \rho\cos\psi$, $y_2 = \rho\sin\psi$). To find the coordinates of point T_2 and distance l by numerical methods, the transcendental equation with respect to angle ψ was solved

$$y_1 - \rho\sin\psi = (x_1 - \rho\cos\psi)\,\mathrm{tg}(\alpha - 2i) , \tag{7}$$

where ρ is specified by expression (3).

Given the position of point T_2 in the (ρ, ψ) coordinates and that of the focus F_d on the arc (r_d, α),

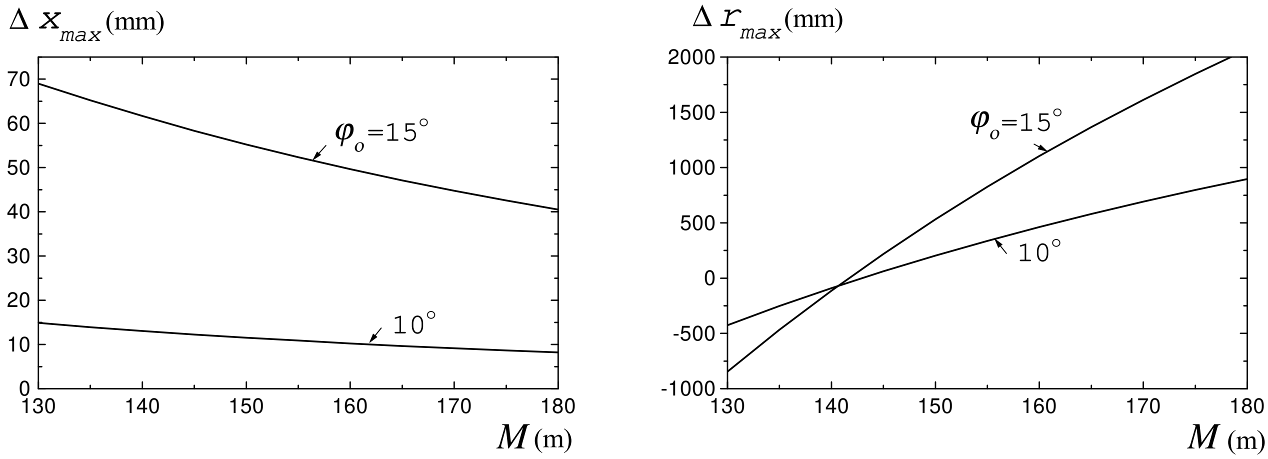

Figure 5: *Maximum longitudinal displacement of the flat mirror elements Δx_{max} and maximum radial shift of the circular reflector elements Δr_{max} versus focal distance M of the circular mirror.*

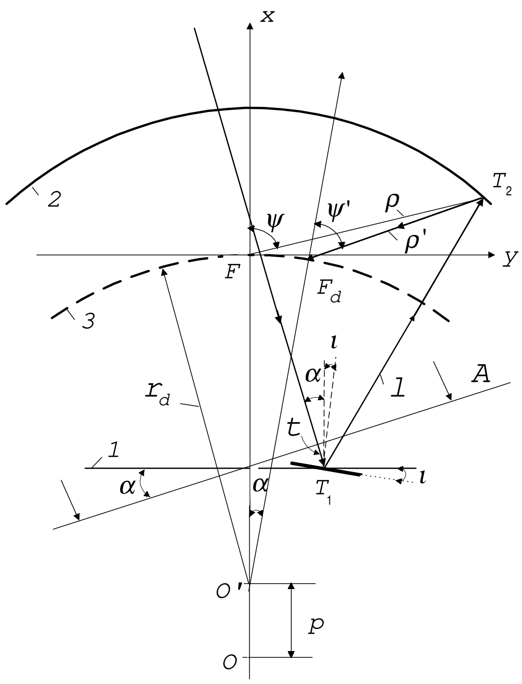

Figure 6: *Ray path in the horizontal plane of the radiotelescope focusing system in the "radio-Schmidt telescope" mode at $H_a \neq 0°$ and $\alpha \neq 0°$.*

little manipulations yield the radius-vector of point T_2 in the coordinate system (ρ', ψ').

The field phase distributions over the aperture thus computed were used to calculate the BP of the radio telescope. Note that the phase distributions have both the linear component and higher order components, cubic, in particular. The value of the latter will affect the level of the side lobes and the value of maximum of the antenna BP. As the calculations have shown, the phase distortions grow with increasing azimuth angle.

Figs. 7, 8 show the BP of the radio telescope at 6 cm under the condition of "radio–Schmidt telescope" in observations at azimuths 0°, 3° and 5° at $\varphi_0 = 10°$ and 15° ($L = 100$ and 150 m). The elevation of the source observed is zero. The aperture of the circular reflector in the calculations was taken equal to $2\varphi_{r\circ} = 30°$ and 40°, respectively ($2\varphi_{r\circ}$ is the angle at which the aperture of the circular reflector is visible from the antenna centre O).

The BP was computed by a standard programme taking into account all features of the RATAN-600 antenna system geometry, proceeding from the BP of the primary feed horn. The algorithms of this programme are presented in part in the paper by Korzhavin (1979). Using the procedure of sequential approximations we would find such a value of p at which the phase variation over the aperture was a minimum at a given azimuthal angle. This parameter defines the position of focus F_d on the arch.

The calculations were performed for two cases. In the former the flat mirror elements are aligned in the curves described by expressions (1, 2) when forming the surface. In this case the mirror elements are displaced longitudinally and rotated in azimuth. Let us name this condition an "exact radio–Schmidt".

In the second case, which is called an "approximate radio–Schmidt", the flat mirror elements are displaced only in longitude with a zero angle of azimuthal rotation. The "Schmidt" correction of the telescope in Nancay is executed in a similar manner.

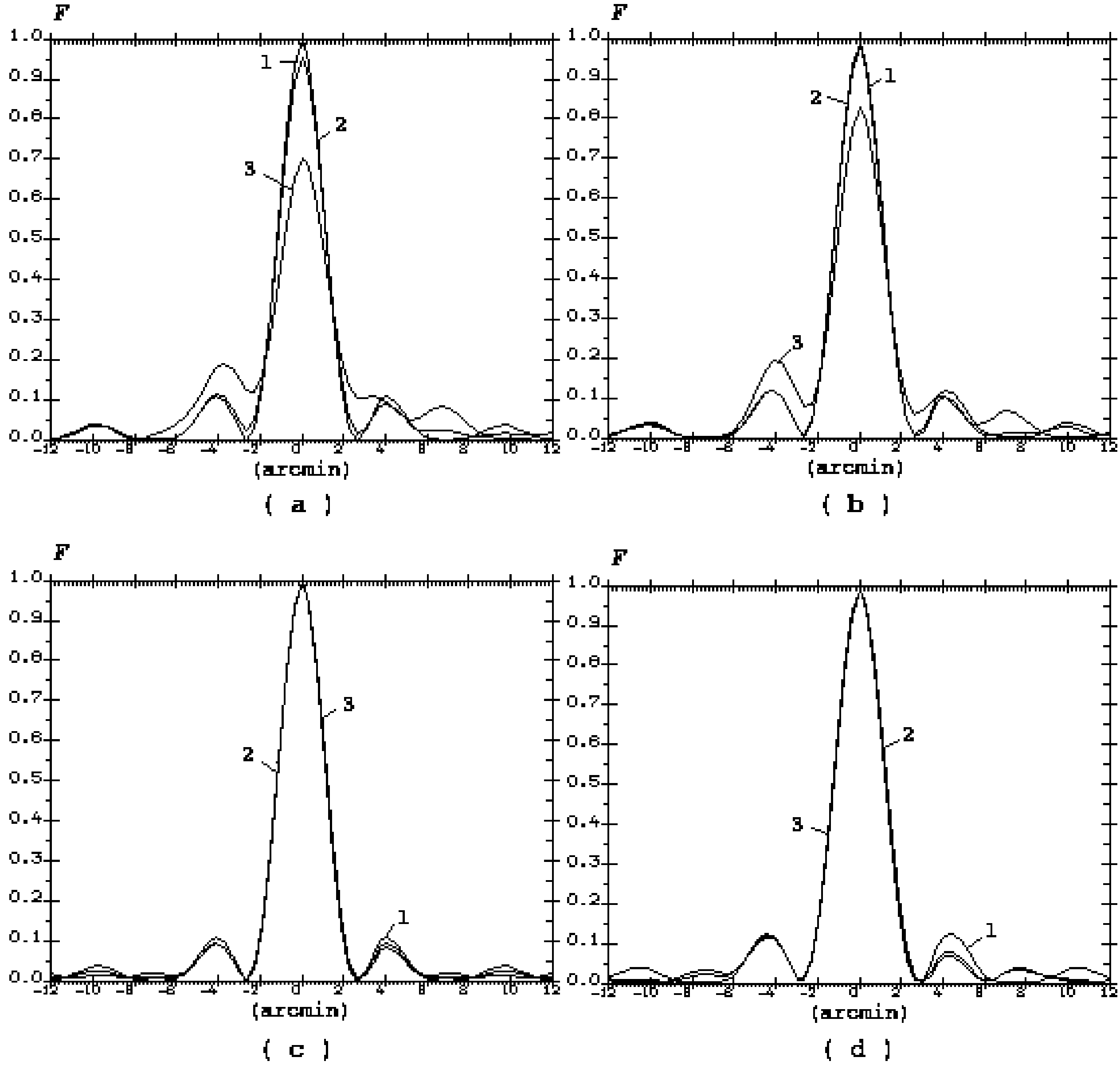

Figure 7: *BP of the RATAN-600 at 6 cm in the mode "radio–Schmidt telescope" for observations in azimuths:* $\alpha = 0^\circ$ *– (1),* $\alpha = 3^\circ$ *– (2),* $\alpha = 5^\circ$ *– (3). (a), (c) – "exact radio–Schmidt", (b), (d) – "approximate radio–Schmidt" (a), (b) – real arch rail track, (c), (d) – optimal arch rail track. Horizontal size of the flat mirror L=100 m (*$\varphi_\circ = 10^\circ$*). Antenna forming angle* $H_a = 0^\circ$.

The curves displayed in Figs. 7, 8 illustrate that in observations with reduced aperture ($L = 100$ m) the BP is distorted only slightly with changing azimuthal angle, the BP being distorted less for the operation mode "approximate radio–Schmidt". In observations with the 150-m aperture we have a formed BP at azimuthal angles ranging from -5° to 5°, which corresponds to about an hour's tracking of the source, the maximum of the BP is, however, being reduced by nearly 40 %. The "exact radio–Schmidt" yields a BP with higher maximum, but within smaller limits (from -3° to 3°). Note that $\alpha = -3^\circ \div 3^\circ$ corresponds to a time of tracking within half an hour at low elevations. The BP exhibited in Figs. 7 (a, b) and 8 (a, b) have been derived for the case where the feed moves on the arch rails that actually exist at RATAN-600. This is the arc of a circle of radius 156 m with the centre at the antenna centre. The distortions of the BP with variation of the azimuthal angle suggest that the arch shape is not optimal. By varying the parameter r_d with a fixed value of α, optimal curvature radii of the arch r_d^{opt} were found, at which the distortions of the BP were a minimum (Fig. 9). As the computations have shown, the shape of the arch

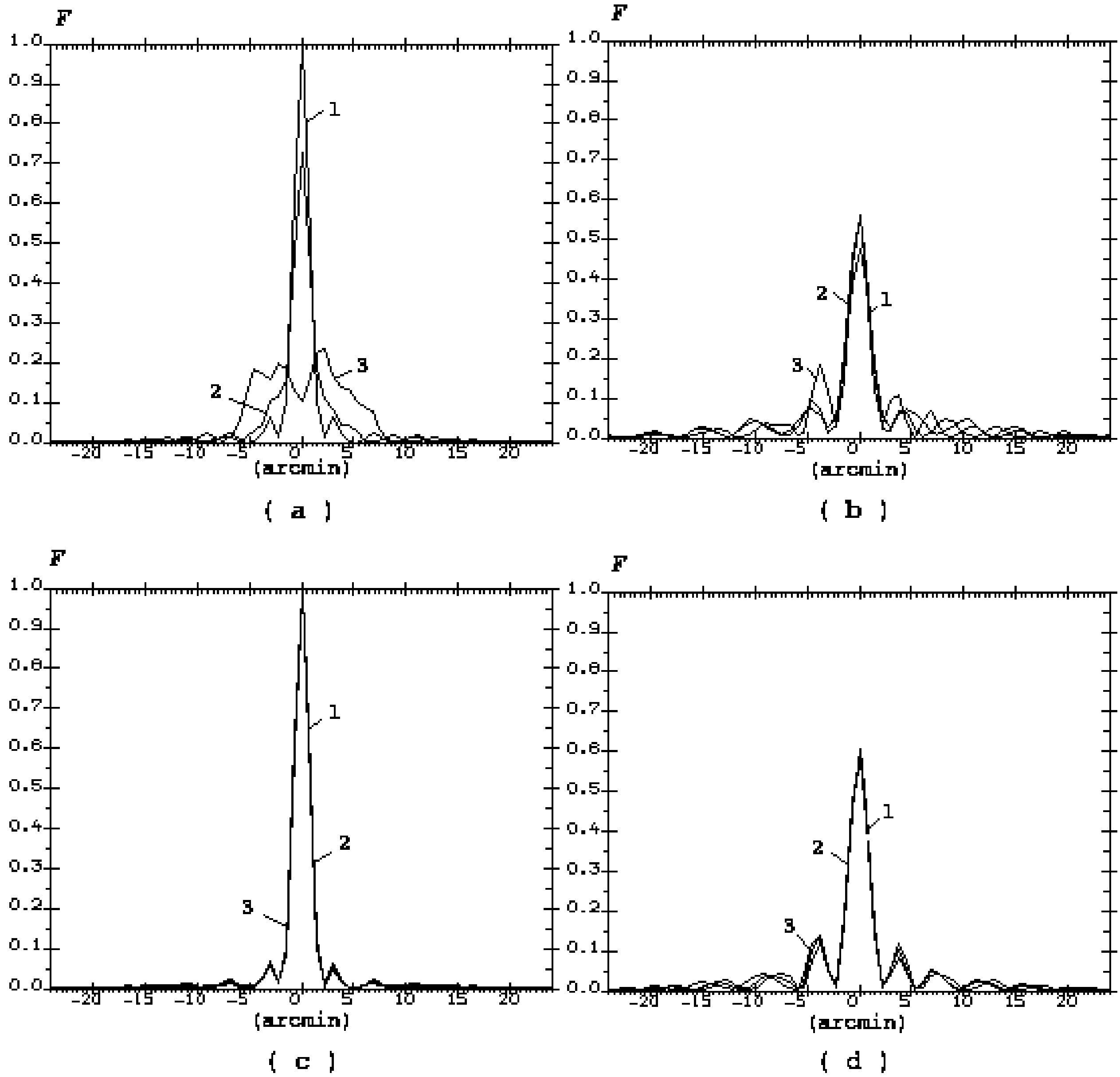

Figure 8: *BP of the RATAN-600 at 6 cm in the mode "radio–Schmidt telescope" for observations in azimuths: $\alpha = 0°$ – (1), $\alpha = 3°$ – (2), $\alpha = 5°$ – (3). (a), (c) – "exact radio–Schmidt", (b), (d) – "approximate radio–Schmidt" (a), (b) – real arch rail track, (c), (d) – optimal arch rail track. Horizontal size of the flat mirror L=150 m ($\varphi_\circ = 15°$). Antenna forming angle $H_a = 0°$.*

on which the feed moves is different for the conditions of "exact" and "approximate radio–Schmidt". Besides, when operated as the "approximate radio–Schmidt", the shape of the optimum arch varies with varying sizes of the aperture (100 m and 150 m). As can be seen from Fig. 9, the optimum arch must be more "straightened" compared with the real one.

In Figs. 7 (c, d) and 8 (c, d) are displayed the BP at 6 cm with the feed moved on the optimum arch. It is seen that under the condition "exact radio–Schmidt" the optimum arch enables tracking of a target for an hour without distortion of the BP both with the 100 m and 150 m apertures. In the mode of "approximate radio–Schmidt" the optimum arch will be effective with the reduced aperture ($L = 100$ m), with $L = 150$ m the BP has a much larger level of side lobes ($12 \div 14$ %) and a lower value of the maximum than in the case of "exact radio–Schmidt", its shape, however, remains practically unchanged. For comparison, in the course of an hour's tracking with the radio telescope in Nancay under the condition of "approximate radio–Schmidt" at 11 cm with the aperture of 200 m the side lobe grows to 12 % (Maghoo, 1990).

Note once again that the computations were per-

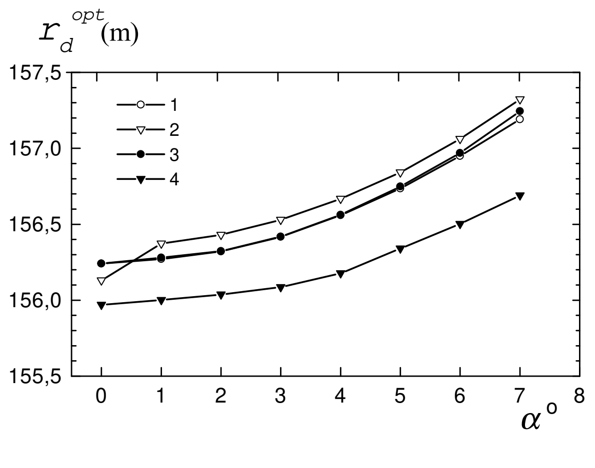

Figure 9: *Dependences of optimum curvature radius r_d^{opt} of the arch rail track on the angle α. "Exact radio–Schmidt" – (1,3), "approximate radio–Schmidt" – (2,4), L=100 m – (1,2), L=150 m – (3,4).*

formed under the assumption the incident wave front makes a zero angle with the horizon, hence the above phase distribution and the BP of the telescope will occur only in observations of sources with a zero elevation. In observing sources with elevations different from zero, the phase distribution over the aperture and the BP of the radio telescope will be different. How great the differences will be and to which elevations the results obtained here are applicable can be assessed only having performed computations of the system for an arbitrary angle of wave front incidence.

3. The design of the antenna system at $H_a \neq 0°$.

In order to obtain a complete picture of the antenna system of RATAN-600 operating as a "radio–Schmidt telescope", calculate the mirror configurations which make it possible to focus the rays incident at an arbitrary angle with the horizon. The azimuthal angle of the wave front equals zero. In this case the condition of absence of spherical aberration is specified by the following expression:

$$\rho + l - x\cos H_a = 2d + q(1 - \cos H_a)\,, \tag{8}$$

which describes the fact that the sum of optical paths from the wave front plane incident at an angle H_a with respect to the horizon to the focus of the system is the same. ρ is the distance of the point the ray intersects the circular mirror from the focus F, l is the distance between the points of intersection of the ray with the mirrors, $x\cos H_a$ is the separation of the incident wave front plane from the flat mirror plane, d is the distance between the vertices of the mirrors, q is the distance of the flat mirror vertex from the focus F. The condition of Abbe sines without coma is

$$y = \sin\psi\,. \tag{9}$$

Taking into account the relationships in the focusing system:

$$x = \rho\cos\psi - l\cos 2i\,,$$
$$y = \rho\sin\psi + l\sin 2i\,,$$
$$\psi = 2i' - 2i\,,$$
$$\mathrm{tg}\, i' = \frac{1}{\rho}\frac{d\rho}{d\psi}\,,$$

and also expressions (8, 9), have the following differential equation

$$\frac{1}{\rho}\frac{d\rho}{d\psi} = \frac{\mathrm{tg}\,\psi/2 - G}{1 + G\mathrm{tg}\,\psi/2}\,, \tag{10}$$

where

$$G = \frac{T - \sqrt{T^2 - B^2(1-\cos^2 H_a)}}{B(1-\cos H_a)}\,,$$

$$T = 2d + q(1-\cos H_a) - \rho(1 - \cos\psi\cos H_a)\,,$$

$$B = (1+\rho)\sin\psi\,.$$

This differential equation was solved by the numerical method of Runge–Kutta with a computer. Programmes were created allowing the following parameters to be computed: x, y, ρ, l, i depending on the focal distance F' of the system, size of the flat mirror L $(2\varphi_\circ)$, wave front incidence angle H_a. The angle H_a for which the system was computed will be called the angle of antenna formation.

The expression for x, l, i:

$$l = \frac{T}{1 + \cos H_a(1-G^2)/(1+G^2)}\,, \tag{11}$$

$$i = \mathrm{arctg}\, G\,, \tag{12}$$

$$x = \rho\cos\psi - \frac{T}{\cos H_a + (1+G^2)/(1-G^2)}\,. \tag{13}$$

Fig. 10 shows how the maximum change in the longitudinal coordinate of the flat mirror, $\Delta x_{max} = x(y) - x(0)$, varies with varying H_a from 0° to 90° for $\varphi_\circ = 10°, 15°$. The shape of the circular reflector does not change with H_a and the relation $\rho(\psi)$ at $H_a = 0°$ holds for it to a high degree of accuracy. It should be noted that, the same as for the case $H_a = 0°$, in the calculation of the shape of the mirrors, optimum values of the paraxial focus of the system were found, at which the longitudinal shifts of the flat mirror were a minimum. The optimum F'_{opt} focus values of the system are different for $\varphi_\circ = 10°$ and $\varphi_\circ = 15°$.

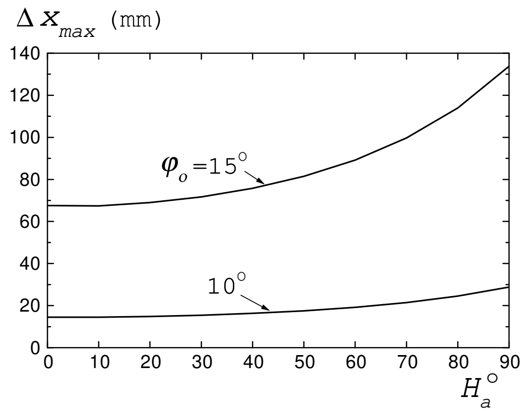

Figure 10: *Maximum longitudinal displacement of the flat mirror panels, Δx_{max}, against antenna formation angle H_a.*

4. The calculation of the phase field distribution in the aperture and BP of the radio telescope in observing at azimuths, $H_a \neq H_i$

Let us calculate the phase field distribution in the antenna aperture, when a source whose elevation H_i is unequal to the angle of antenna formation and the azimuth of the target $\alpha \neq 0°$. The elements of the flat and circular reflectors are set in the curves specified by expressions (9, 10, 12, 13). These were derived for the case where the flat wave front incident on the first mirror makes an angle H_a with the horizon and $\alpha = 0°$ (transit of the source across the meridian) with the antenna system axis.

The circular antenna elements are set in a rigorously upright position to form a quasicircular cylinder with a vertical generant. The flat mirror elements are inclined with respect to the horizon through an angle $H_u/2$. In the case where the azimuth α of a source is different from zero, the angles H_u and H_i are related as follows:

$$\operatorname{tg} H_u = \frac{\operatorname{tg} H_i}{\cos\alpha} \,. \tag{14}$$

The feed is displaced on the arch rails from the system axis and the location of a new focus F_d in the polar coordinate system whose centre O' is on the system axis at a distance p from the centre O of the circle is given by the coordinates (r_d, α) (Fig. 6). The sum S of the optical paths from the focus F_d to the plane AA normal to the wave front incident at the angle H_i with the horizon is

$$S = t_h + l + \rho' \,, \tag{15}$$

where t_h is the distance of the plane going through the vertex of the flat mirror from point T_1 on the flat mirror, l is the distance between points T_1 and T_2 on the surface of the flat and circular mirrors, ρ' is the distance of point T_2 on the flat mirror from the focus F_d or the radius-vector of point T_2 in the polar coordinate system (ρ', ψ'). The straight line A in Fig. 6 shows the line of intersection of the plane AA with the horizontal plane.

Calculate the phase field distribution over the aperture, expressed in a linear measure, $\Delta S(\psi') = S(\psi') - S(0)$.

$$t_h = t\cos H_i = (y_1 \sin\alpha - x_1 \cos\alpha)\cos H_i \,. \tag{16}$$

The coordinates x_1, y_1 of the flat mirror and the angle i of the tangent line to the curve at point T_1 with the axis y were computed by formulae (9, 10, 12, 13) for the angle H_a of antenna formation.

To find the coordinates of point T_2, at which the ray intersects the circular reflector, and the value of l, the transcendental equation with respect to the angle ψ was solved:

$$y_1 - \rho \sin\psi = (x_1 - \rho\cos\psi)\operatorname{tg}(\alpha_0 - i) \,, \tag{17}$$

where α_0 is the azimuthal angle of the reflected ray

$$\sin\alpha_0 = \sin(\alpha - i)\cos H_i \,. \tag{18}$$

When solving this equation for ρ, expression (3) derived with $H_a = 0°$ has been used, since, as has been noted, the relationship $\rho(\psi)$ does not depend on H_a.

Knowing location of point T_2 in the coordinate system (ρ, ψ) and the position of the F_d focus on the arc, (r_d, α), a little manipulation yields the vector-radius of point T_2 in the system (ρ', ψ').

In Figs. 11 and 12 are displayed the results of field phase calculations in the antenna aperture for the modes "exact" and "approximate radio–Schmidt" at $\varphi_\circ = 10°, 15°$, $\alpha = 0°, 3°, 5°$ and different H_a. On the ordinate axis are laid off the values of maximum phase difference ΔS_{max} (in mm), the abscissa is the plot of the elevation H_i of the observed source. The linear component of the phases was preliminarily subtracted, since it does not cause distortion of the BP but only its turn. Thus, these curves can be used to estimate the maximum phase error when observing a source at H_i, when the antenna is set for an angle $H_a \neq H_i$.

From examination of the curves presented in Figs. 11, 12 the following conclusions can be drawn.

As it was to be expected, the cophasal field distribution in the antenna aperture ($\Delta S = 0$) (or close to cophasal — in the case of "approximate radio–Schmidt") is realized in observations of sources whose elevation is equal to the angle of antenna formation and $\alpha = 0°$, as well as at azimuth angles different from zero, for $H_a > 70°$. For the antenna system designed for small and medium angles, the maximum phase error increases with growing azimuthal angle.

In the H_a range from 0° to 20° the relations $\Delta S_{max}(H_i)$ are only slightly different. So, with the

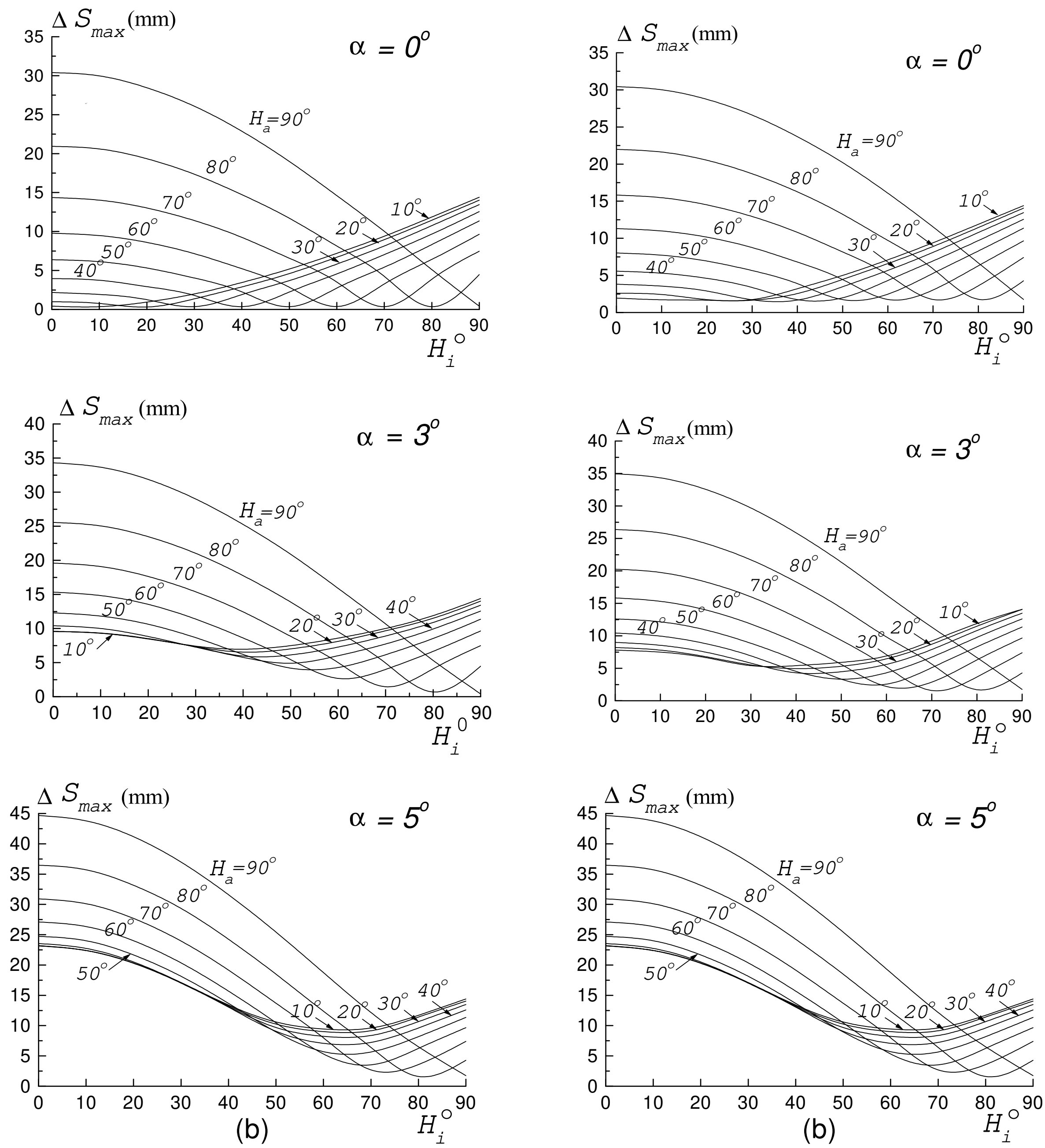

Figure 11: *Maximum phase difference in the aperture ΔS_{max} as a function of source elevation H_i at $\alpha = 0°$, $3°$, $5°$ and different antenna formation angles H_a. The horizontal size of the flat mirror $L = 100\,m$ ($\varphi_\circ = 10°$). (a) – "exact radio–Schmidt", (b) – "approximate radio–Schmidt".*

antenna computed for $H_a = 0°$ it is possible to observe targets the elevation of which does not exceed $20°$.

Comparing the relations $\Delta S_{max}(H_i)$ at $\varphi = 10°$ and $15°$, one can state that their character preserves with changing aperture sizes, and only the absolute value of the maximum phase difference in the aperture changes.

At last, the maximum phase errors in the mode "approximate radio–Schmidt" at azimuthal angles of $3° \div 5°$ and with a reduced aperture ($\varphi_\circ = 10°$) are smaller than or equal to ΔS_{max} under the condition "exact radio–Schmidt". The same can be stated for the 150 m aperture ($\varphi_\circ = 15°$) for H_a below $60°$. This is why in further calculations of the radio telescope BP, primary attention was given to the mode "approximate radio–Schmidt".

For a more detailed study, BP of RATAN-600 were

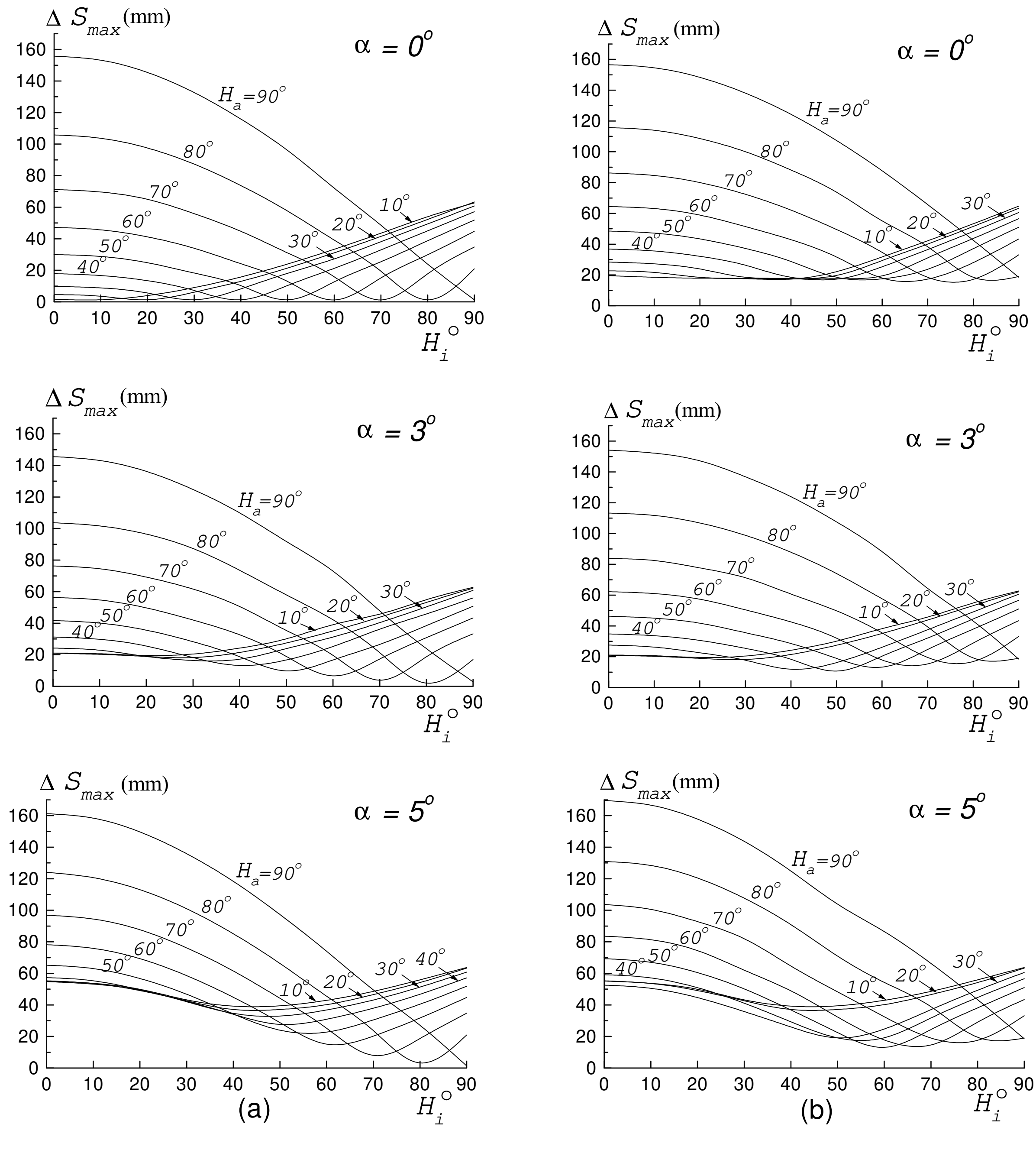

Figure 12: *Maximum phase difference in the aperture ΔS_{max} as a function of source elevation H_i at $\alpha = 0°$, $3°$, $5°$ and different antenna formation angles H_a. The horizontal size of the flat mirror L = 150 m ($\varphi_\circ = 15°$). (a) – "exact radio–Schmidt", (b) – "approximate radio–Schmidt".*

computed at the wavelengths 1 cm, 2 cm, 4 cm, 8 cm, 12 cm, 16 cm and 32 cm for the apertures of 100 m and 150 m ($\varphi_\circ = 10°, 15°$, respectively).

Figs. 13, 14 show the relationships between the maximum BP and the elevation H_i of the source observed for three angles of antenna formation, H_a = 0°, 30°, 45°. Curves (1, 2, 3) show the relationships under the condition "approximate radio–Schmidt", curve (4) in the "exact radio–Schmidt" mode at $H_a = 30°$.

The BP of the radio telescope operated as the "approximate radio–Schmidt" at 2 cm, 4 cm, 8 cm, 16 cm and 32 cm in observing sources with $H_i = 10°, 20°, 30°, 40°, 50°, 60°, 70°$ are shown in Figs. 15, 16. The angle of antenna formation H_a = 30°. The calculations were performed for the available arch rails.

The analysis of the relationships allows us to claim

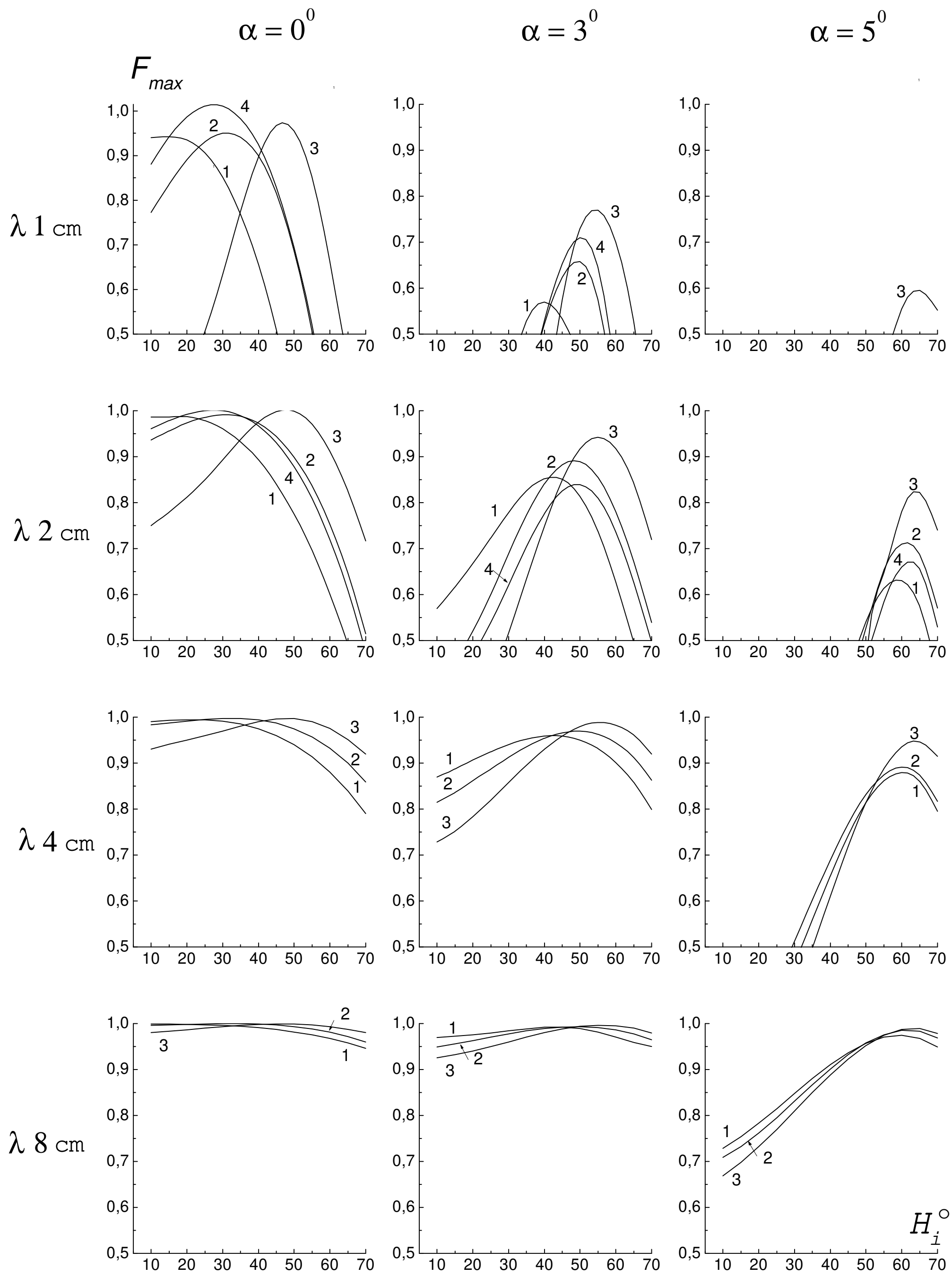

Figure 13: *Maximum RATAN-600 BP versus source elevation H_i at antenna formation angles $H = 0°$ – (1), $30°$ – (2, 4), $45°$ – (3). "Approximate radio–Schmidt" – (1, 2, 3), "exact radio–Schmidt" – (4). The flat mirror size L=100 m ($\varphi_\circ = 10°$).*

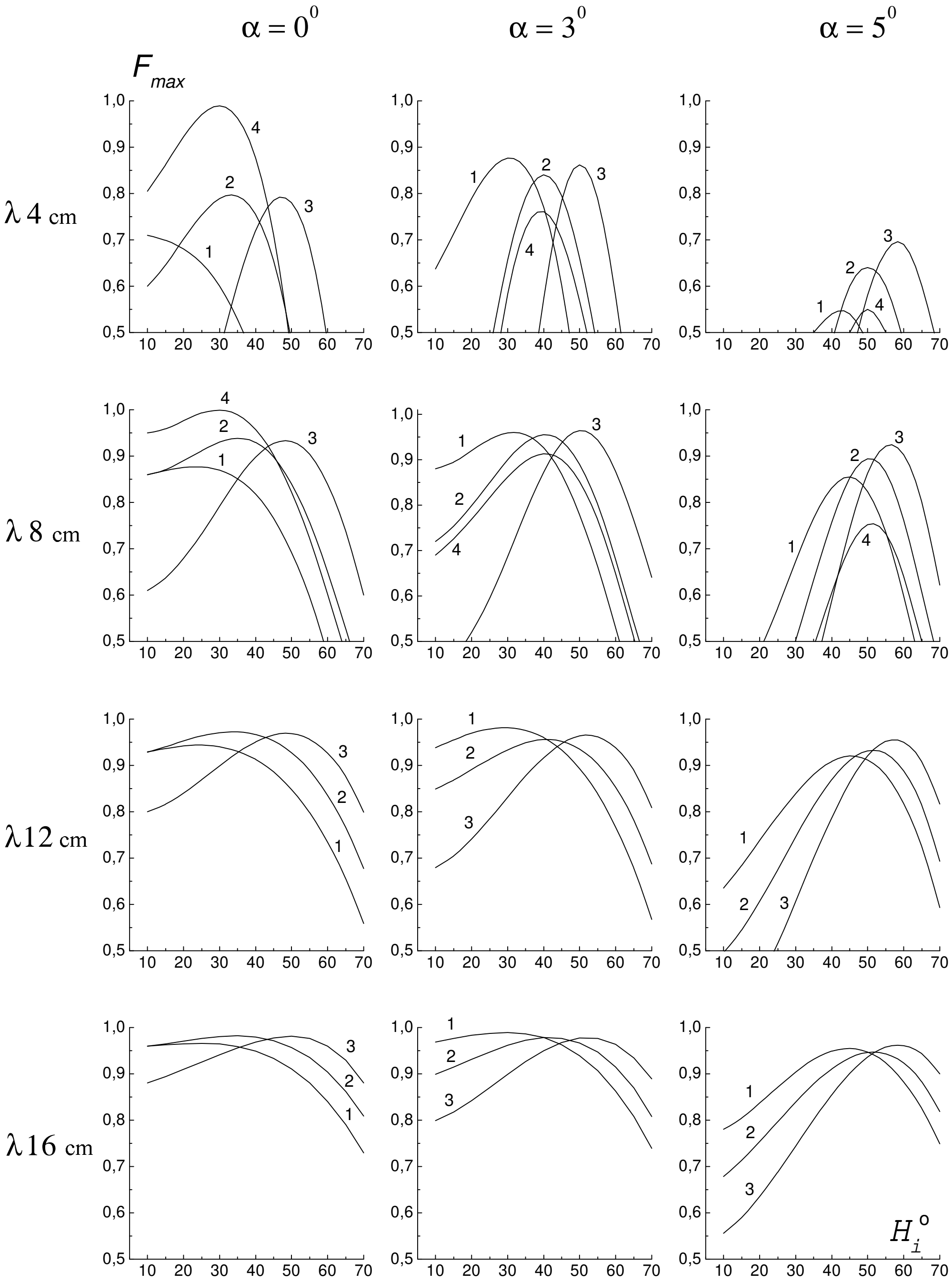

Figure 14: *Maximum RATAN-600 BP versus source elevation H_i at antenna formation angles $H = 0°$ – (1), $30°$ – (2, 4), $45°$ – (3). "Approximate radio–Schmidt" – (1, 2, 3), "exact radio–Schmidt" – (4). The flat mirror size L=150 m ($\varphi_\circ = 15°$).*

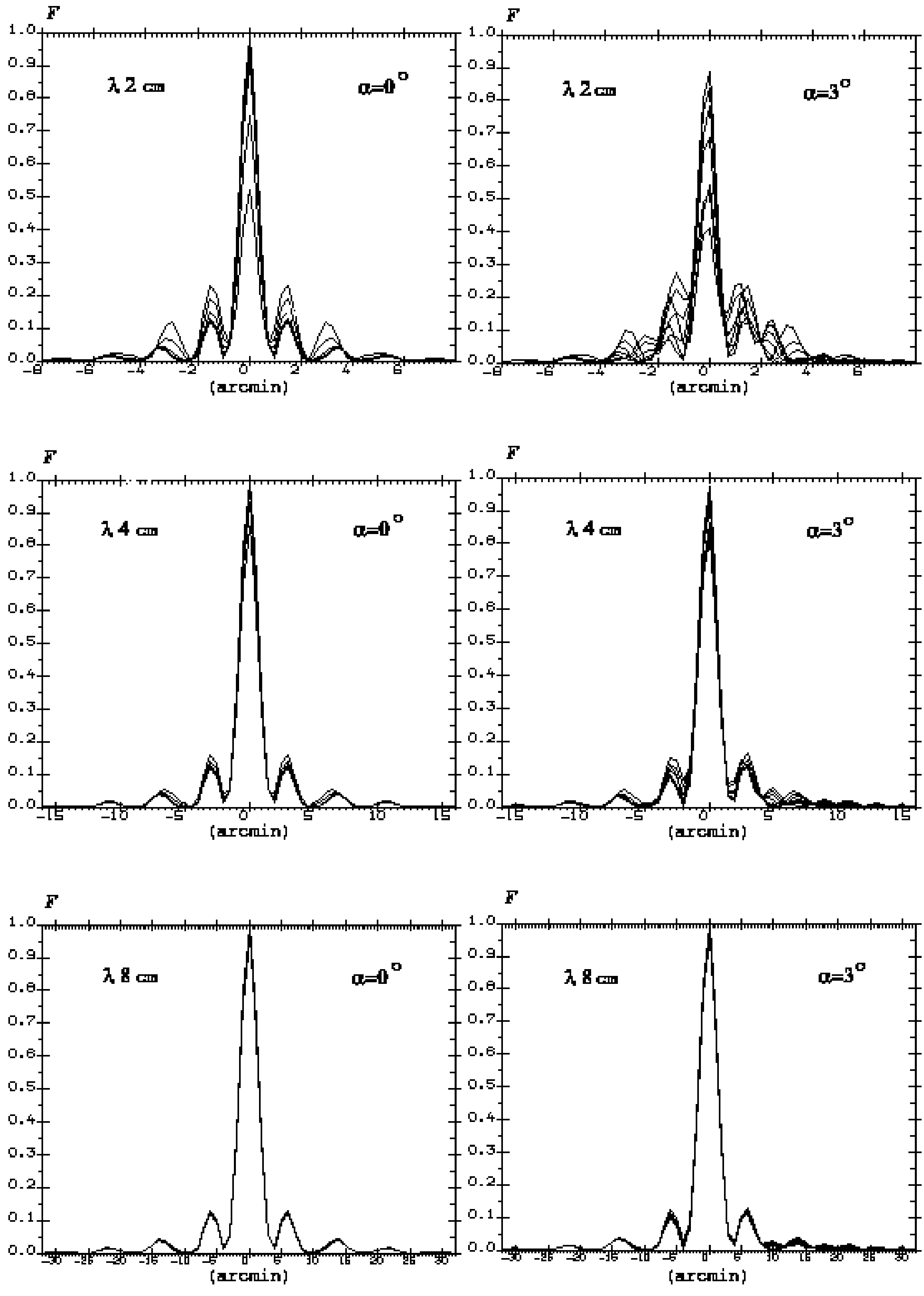

Figure 15: *The BP of RATAN-600 under the condition of "approximate radio–Schmidt" at waves $\lambda = 2\,cm$, $4\,cm$, $8\,cm$ at source elevations $H_i = 10°$, $20°$, $30°$, $40°$, $50°$, $60°$, $70°$ and $\alpha = 0°$, $3°$. The antenna formation angle $H_a = 30°$, L=100 m ($\varphi_\circ = 10°$).*

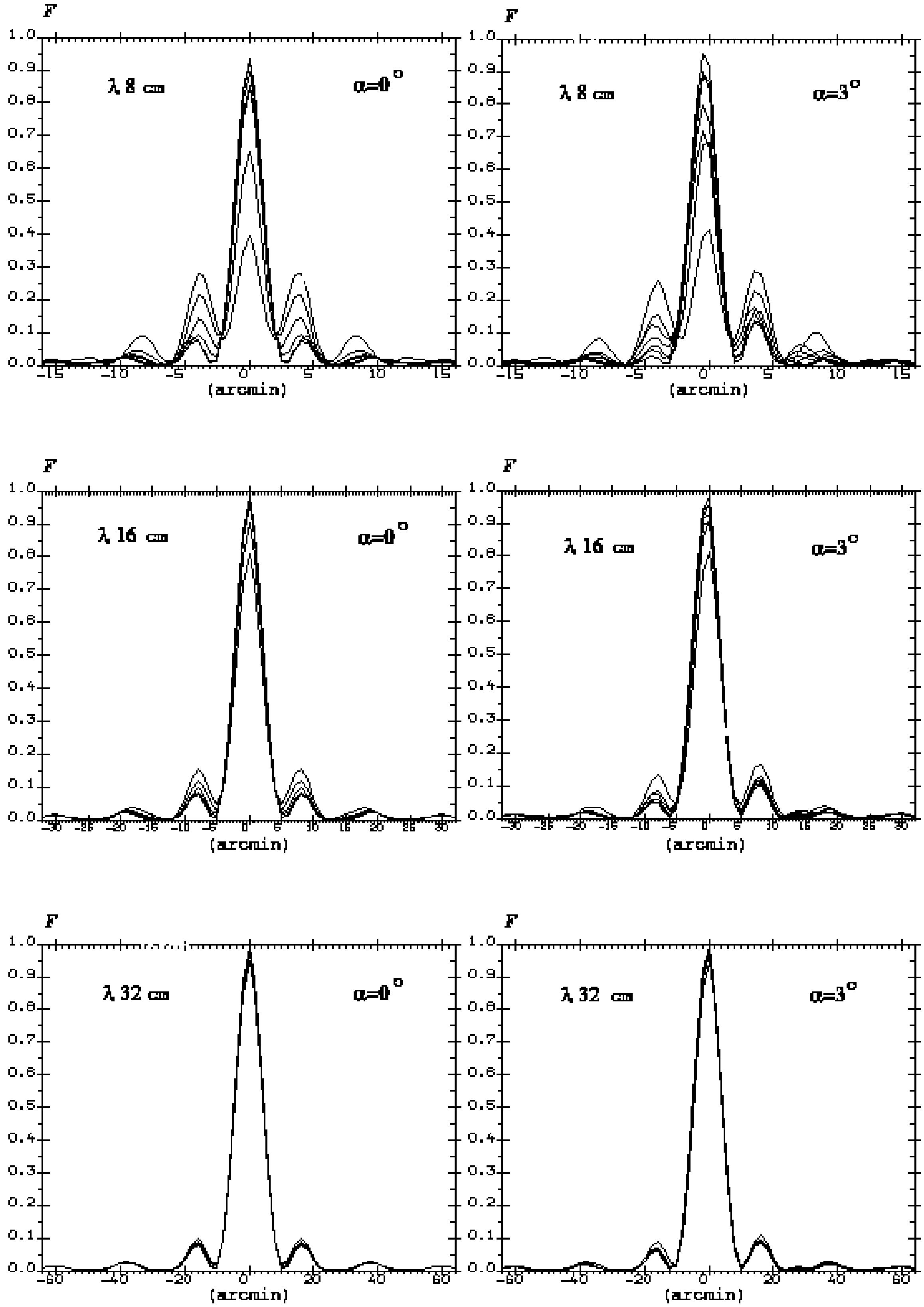

Figure 16: *The BP of RATAN-600 under the condition of "approximate radio–Schmidt" at waves $\lambda = 8\,cm$, $16\,cm$, $32\,cm$ at source elevations $H_i = 10°$, $20°$, $30°$, $40°$, $50°$, $60°$, $70°$ and $\alpha = 0°$, $3°$. The antenna formation angle $H_a = 30°$, $L{=}150\,m$ ($\varphi_\circ = 15°$).*

that with the aid of the "South+Flat", a focusing system, the so-called "radio–Schmidt telescope", can be constructed, that will enable long tracking (from half an hour to one hour) of a source with no essential distortions of the BP in a wide range of wavelengths and source elevations. The "approximate radio–Schmidt" mode is more flexible as compared to the "exact radio–Schmidt". The latter has an advantage only at short wavelengths ($\lambda \cong 1$ cm at $\varphi_\circ = 10°$, $\lambda \cong 2$ cm at $\varphi_\circ = 15°$) and also in observations at azimuths close to zero.

As for the choice of the antenna formation angle, the range of angles H_a from 0° to 30° seems to be the most optimum. At these angles the range of elevations at which one can observe sources with permissible losses (the reduction of maximum BP by 20–30 % may be taken as a criterion) is a maximum. The final choice of the antenna formation angle should, however, be made proceeding from a particular astrophysical task. The relations presented show that the reduced aperture ($L = 100$ m) mode is for shorter wavelengths. The 150 m aperture provides a larger effective area, but it can be employed at waves no shorter than 4 cm.

For the tasks that do not need long tracking of a source but only signal accumulation at one declination, one can use the "exact radio–Schmidt" with an antenna aperture up to 170–200 m computed for a specified elevation $H_a = H_i$. A wide field of view is achieved through placing a multi-element receiver (array) along the focal plane of the secondary mirror (focal line of the feed). This condition is to be discussed in a separate paper.

5. Conclusions

As a result of the research carried out, formulae have been derived and programmes have been written to calculate the surface of the flat and circular reflectors of the radio telescope RATAN-600 to be operated as a double-mirror aplanatic system — the so-called "radio–Schmidt telescope".

The phase distributions of the field in the aperture and BP of the radio telescope under the condition of "exact" and "approximate radio–Schmidt telescope" have been computed for the case of zero angle of antenna formation and for the case this angle is other than zero and does not coincide with the elevation of the source being observed. The computations performed have made it possible to optimize the parameters of the system discussed, in particular, the paraxial focal distance of the telescope, the curvature of the arch rails that carry the feed, the range of angles of antenna formation, and the angles of source observation. The computations done for a wide range of angles and wavelengths provide data on the basis of which a mode of operation of the radio–Schmidt telescope can be chosen, depending on an astrophysical tasks to be performed.

The "approximate radio–Schmidt" with the currently existing arch rails has been shown to enable tracking of sources for an hour with the 150 m aperture, at wavelengths longer than 4 cm. The optimization of the arch rails will permit long-time observations at wavelengths up to 1 cm to be made.

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